A Consequentialist Theory of Morality

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Abstract

This article explains a two consequentialist theories of morality, that is to say theories of morality that are based on the consequences of actions. This article assumes a background in the axiomatic arithmetic, set theory and probability theory that can be found in certain Stage 2 university mathematics and computer science courses.

1 What is morality?

Morality is the subject of the general principles concerned with the way that conscious beings such as people should interact with each other. One theory of morality that I will present in this article uses probabilities so first we need to revise some elementary probability theory.

2 Axiomatic definition of probability

The expression $P(A)$, the probability of an event A is a number assigned to the event A in the sample space Ω such that:

Axiom 1 *For any event A, $P(A) \geq 0$.*

Axiom 2 $P(\Omega) = 1$

Definition 1 *Two events A and B are* mutually exclusive *if and only if $P(A \cap B) = \emptyset$. A collection of events $S = A_1, A_2, A_3, \cdots$ are mutually exclusive if and only if $P(A_i \cap A_j) = \emptyset$ for all $i, j \in \{1, 2, 3, \cdots, |S|\}$.*

Axiom 3 *For any countable collection $A_1, A_2, A_3, \cdots$ of events that are mutually exclusive: $P(A_1 \cup A_2 \cup A_3 \cup \cdots) = P(A_1) + P(A_2) + P(A_3) + \cdots$.*

Lemma 1 *Let $A, B \subseteq \Omega$. Then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.*

Proof Firstly note that $(A \backslash B) \cap (A \cap B) = \emptyset$ and $(A \backslash B) \cup (A \cap B) = A$. Therefore:

$$\begin{array}{rcll} P(A \backslash B) + P(A \cup B) & = & P((A \backslash B) \cup (A \cap B)) & \text{by Axiom 3} \\ & = & P(A) & \text{since } (A \backslash B) \cup (A \cap B) = A \dagger. \end{array}$$

Secondly note that $(B \backslash A) \cap (A \cap B) = \emptyset$ and $(B \backslash A) \cup (A \cap B) = B$. Therefore

$$\begin{array}{rcll} P(B \backslash A) + P(A \cap B) & = & P((B \backslash A) \cup (A \cap B)) & \text{by Axiom 3} \\ & = & P(B) & \text{since } (B \backslash A) \cup (A \cap B) = B \ddagger \end{array}$$

Thirdly let $X_1 = A \backslash B$, $X_2 = A \cap B$ and $X_3 = B \backslash A$. Then X_1,X_2 and X_3 are mutually exclusive and $A \cup B = X_1 \cup X_2 \cup X_3$.

$$\begin{array}{rcll} P(A \cup B) & = & P(X_1) + P(X_2) + P(X_3) & \text{by Axiom 3} \\ & = & P(A \backslash B) + P(A \cap B) + P(B \backslash A) & \text{by definition of } X_1, X_2 \text{ and } X_3. \end{array}$$

Therefore:

$$\begin{array}{rcll} P(A \cup B) + P(A \cap B) & = & P(A \backslash B) + P(A \cap B) + P(B \backslash A) + P(A \cap B), & \\ & & \text{adding } P(A \cap B) \text{ to both sides} & \\ & = & P(A) + P(B) \text{ by } \dagger \text{ and } \ddagger. & \end{array}$$

Subtracting $P(A \cap B)$ from both sides yields the result. ∎

Lemma 2 *Suppose that $A \supseteq B$. Then $P(A) \geq P(B)$*

Proof Suppose that $A \supseteq B$. Then $A = B \cup (A \backslash B)$ and $B \cap (A \backslash B) = \emptyset$. Therefore

$$\begin{array}{rcll} P(A) & = & P(B) + P(A \backslash B) & \text{by Axiom 3} \\ & \geq & P(B) & \text{since } P(A \backslash B) \geq 0 \text{ by Axiom 1.} \end{array}$$ ∎

3 Two theories of morality

3.1 Potential harm

A natural way to define morality is by the consequences of actions, specifically the probability of an individual $i \in \wp$ (where $\wp$ denotes the set of all people) receiving harm in the form of retribution from another individual $j \in \wp$. Let us define $P(H_i)$ as the probability of person i receiving harm. Furthermore let us define $P(H_{i,j})$ as the probability of person i receiving harm from person j. With these two definitions in place then the following result holds:

$$P(H_i) = P(\cup_{j \in \wp} H_{i,j})$$

where $i \in \wp$. Later on in this article I will make the notion of "harm" less vague and more logically rigorous.

3.2 Legacy

Another way to define morality is by the extent of one's legacy. It is related to the above definition of morality in terms to potential harm, but at the same time also different. For example a person with a high $P(H_i)$ from advocating a controversial political policy does not correspond to the size of that person's legacy. If someone is assassinated it is likely that their legacy will be a strong one despite the high value of $P(H_i)$. For example the former US president Abraham Lincoln[1] wanted most to be remembered as a good person and to have his life story retold by future Americans. Lincoln's assassination at the moment of his greatest triumph, the preservation of the in the light of the secessionist Confederate States of America caused his legacy to be better than his wildest imaginations. Having a good legacy is a way to perpetuate ourselves in the form of positive memories of ourselves.

4 Calculating $P(H_i)$

Definition 2 *Two events A and B are* independent *if and only if $P(A \cap B) = P(A) * P(B)$.*

To calculate $P(H_i)$ we need an additional assumption that the set $\wp$ is of the form $\{1, 2, 3, \cdots, n\}$ where n is a natural number so that the set is a finite subset of the natural numbers. This can be done without any loss of generality because the total number of people that have ever lived is a finite set and an arbitrary set $\wp$ can be bijectively mapped into such a subset of the natural numbers.

Theorem 1 *Assuming that the events $H_{i,j}$ and $H_{i,k}$ are independent for all $i, j, k \in \wp$, then $P(H_i) = P(\cup_{j=1}^{n} H_{i,j}) = z_n$ where*

$$\begin{aligned} z_0 &= 0 \\ z_n &= z_{n-1} + P(H_{i,n}) - z_{n-1} * P(H_{i,n}) \text{ for } n \geq 1 \end{aligned}$$

[1] `http://en.wikipedia.org/wiki/Abraham_Lincoln`

Proof This relationship can be proved using mathematical induction. The case $n = 0$ is self evident. For the inductive step, assume the equation is true for $n \geq 0$. Then

$$\begin{aligned}
LHS_{n+1} &= z_{n+1} \\
&= P(\cup_{j=1}^{n+1} H_{i,j}) \text{ by definition of } z_{n+1} \\
&= P(\cup_{j=1}^{n} H_{i,j} \cup H_{i,n+1}) \text{ expanding the union} \\
&= P(\cup_{j=1}^{n} H_{i,j}) + P(H_{i,n+1}) - P((\cup_{j=1}^{n} H_{i,j}) \cap H_{i,n+1}) \text{ by Lemma 1} \\
&= P(\cup_{j=1}^{n} H_{i,j}) + P(H_{i,n+1}) - P(\cup_{j=1}^{n} H_{i,j}) * P(H_{i,n+1}) \\
&\quad \text{since } H_{i,j} \text{ and } H_{i,k} \text{ are independent for all } i, j, k \in \wp \text{ *} \\
&= z_n + P(H_{i,n+1}) - z_n * P(H_{i,n+1}) \text{ by definition of } z_n \\
&= RHS_{n+1} \quad \blacksquare
\end{aligned}$$

* Note that the definition of a collection of independent events needs to be changed before this step follows logically from the previous step.

5 Some deductions about $P(H_i)$ and $P(H_{i,j})$

Theorem 2 *$P(H_i) \leq \epsilon$ implies $P(H_{i,j}) \leq \epsilon$.*

Proof I shall prove the contrapositive. Suppose that $P(H_{i,j}) > \epsilon$. Then

$$\begin{aligned}
P(H_i) &= P(\cup_{k \in \wp} H_{i,k}) \quad \text{by definition of } P(H_i) \text{ and } P(H_{i,k}) \\
&\geq P(H_{i,j}) \quad \text{by Lemma 2, since } \cup_{k \in \wp} H_{i,k} \supseteq H_{i,j} \\
&> \epsilon \quad \blacksquare
\end{aligned}$$

Theorem 3 *$P(H_i) < \epsilon$ implies $P(H_{i,j}) < \epsilon$.*

Proof Similar to the proof of the previous theorem.

Theorem 4 *$P(H_i) > \epsilon$ by itself affords no police protection unless you have a job that comes with police protection such as the leader of a country.*

Theorem 5 *If $P(H_{i,j}) > \epsilon$ and $P(H_{j,i}) < \epsilon$, then person i should terminate their relation to person j and either not communicate or communicate through the medium of lawyers. Some examples:*

- *If person j knows the home address of person i and person i doesn't know the home address of person j, then (in the absence of other mitigating factors) person i should not engage with person j.*
- *If $P(H_{i,j}) > \epsilon$ and $P(H_{j,i}) = 0$ then person i should not make eye contact with person j.*

Theorem 6 *If $P(H_{i,j}) > \epsilon$ and $P(H_{j,i}) > \epsilon$, then persons i and j should mutually terminate their relations to the other person and either not communicate or communicate through the medium of lawyers.*

Theorem 7 *To ensure that $P(H_i) \leq \epsilon$, then by Theorem 2 and Theorem 5, for person i to have a lifetime relationship (in the absence of lawyers) with other persons needs to ensure that $P(H_{i,j}) \leq \epsilon$ for all $j \in \wp$. As will be explained later this requires honesty. Therefore to live outside of the protection of lawyers you must be honest. Therefore Bob Dylan's lyric*[2] "to live outside the law you must be honest" *holds true. Note that for this result to obtain the above definition of living outside the law must apply.*

6 Some general principles of morality

The best way to teach morality is to serve as an example to others of one's own kind, rather than by preaching specific moral values. However over the course of my life I have discovered some general principles of morality that relate to $P(H_i)$ and one's legacy. Here they are (in alphabetical order)

- *Charity / Philanthropy* Giving things away for free or at low cost to serve humanity. Examples include giving away your time, money and knowledge to serve humanity. These practises are a good way to balance out any wrongs that you do. For example this probably explains why Bill Gates is such a philanthropist.

- *Honesty*: Generally speaking, tell the truth with the exception of (where necessary) non-hurtful white lies.

- *Humility / Modesty.* To not present oneself as superior to something that you are not. The philosopher Friedrich Nietzsche[3] described humility as a false virtue[4], only applicable to "losers". Nonetheless lack of humility can affect ones $P(H_i)$ and therefore is relevant to all of us. For example for me to claim that I am the greatest person that ever lived when I clearly am not is an example of lack of humility.

- *Karma*: If you do something bad or otherwise have $P(H_{i,j}) > \epsilon$, then you need to do something good to balance it out. It is honest to keep your badness a secret because you are simply omitting to mention it

[2] `http://en.wikipedia.org/wiki/Absolutely_Sweet_Marie`
[3] `http://en.wikipedia.org/wiki/Friedrich_Nietzsche`
[4] `http://en.wikipedia.org/wiki/Humility`

and therefore it is a non-hurtful white lie. However, your goodness can be put on your CV, for reasons that will be explained later.

- *Knowledge.* Adding to the body of human knowledge is inherently a good thing. Even knowledge of how to create morally questionable objects like weapons of war might have a positive peaceful use and therefore all knowledge (no matter what kind) is good. An example of bad technology having a good use is nuclear weapons. They could be used to prevent an asteroid from hitting the Earth and obliterating life on the planet.

- *Legal and Professional Righteousness.* To behave in accordance with the rules of legal and professional conduct. Not hiring someone who applies for a job can invoke $P(H_{i,j}) > \epsilon$ and therefore by Theorem 2, $P(H_i) > \epsilon$, despite the fact that not hiring someone for a job is a completely moral action under commonly used conceptions of morality.

- *Love*: Where possible, you should prefer praise to deprecation, especially when the subject of deprecation is a person. When something is bad then it might be best to either omit to mention it or to rephrase the deprecatory (destructive) criticism into something constructive (praise). Most criticism can be re-expressed as praise, when viewed from a different point of view. For example criticism of Bill Gates' fearsome tactics for eliminating his competition can be re-expressed as praise for his success in dominating the market. Notwithstanding this, when something really is bad in an important way it is necessary to take a stand against it. Examples include: racism, sexism, Hitler and the Nazis. An example of lack of love is laughing at someone else's misfortune.

Some people cannot preach general moral values because they are not honest. In this case they can only preach applied moral values (i.e. ethical or political) or by (as mentioned above) setting an example to others.

7 Personal Websites and $P(H_i)$

A personal Website presents the case for not being harmed when $P(H_i) > \epsilon$, and therefore it is rational to put a positive spin on everything you put on your Website. Specifically omitting certain bad information is a non-hurtful white lie and therefore not wrong.

8 $P(H_i)$ and different races and sexes

If you are a person i of a different race another person j, then in some people j the value of $P(H_{i,j})$ is smaller then for people j who are the same race as i. Women are the gate-keepers of sex, meaning that if a women wants sex, she just has to open the legs to her vagina and a penis will soon come along, whereas if a man wants sex he has to have a good job, a good car, a good house and so on to attract a female mate. As women are the gate-keepers of sex, women in positions of power have lower $P(H_{i,j})$ than men in equivalent roles, because they can theoretically offer sex to men who have a probability of harming that woman greater than ϵ. This offer of sex can partially offset the wrongs in terms of $P(H_{i,j})$ invoked by that man to that woman. This explains why the jobs of secretaries in organisations are invariably taken by women. For this reason, the future of women in power is a good one, and it is likely that in the future all positions of power will be taken by women.

9 Wearing a tie

Men with a significant $P(H_i)$ will wear a tie to reduce their $P(H_i)$. As women are the gate keepers of sex, they do not need to wear a tie.

10 Shaking hands

Men to men's handshakes are strong if both men have had sex with a woman. Women to men's handshakes are less strong, since there is a probability that the woman could have sex with the man.

11 Explanation of the concept of "harm"

The ultimate harm is death, so by $P(H_i)$, I mean the probability of death for person i and similarly for $P(H_{i,j})$.

12 The problem with Kant's categorical imperative

The problem with Kant's *categorical imperative* is that different moralities apply to different people. Under the potential harm conception of morality this result obtains. *Underlings* are people who are under your control,

directly or indirectly. The value for $P(H_{i,j})$ varies with the number of underlings that a person has because people love to gossip about people who have lots of underlings. Therefore people with a lot of underlings such as leaders of cities, states and countries or university lecturers will have a higher value for $P(H_{i,j})$ and by Theorem 2, $P(H_i)$. Therefore such people have to be more righteous in their conduct than other people because of karma.